

CMB constraints on non-thermal leptogenesis

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Leptogenesis is at the heart of particle cosmology which requires physics beyond the Standard Model. There are two possibilities of realizing leptogenesis; thermal and non-thermal. Both are viable given the scale of inflation and the constraint on the reheat temperature. However non-thermal leptogenesis can leave its imprint upon cosmic micro wave background radiation. In this paper we will discuss cosmological constraints on non-thermal leptogenesis scenarios within supersymmetry.

I. INTRODUCTION

Baryogenesis and neutrino oscillations are the two fronts which naturally evoke physics beyond the electroweak Standard Model (SM). Both Big bang nucleosynthesis and current WMAP data suggest that the baryon asymmetry is of the order of one part in 10^{10} [1], while the solar neutrino experiments suggest $\Delta m_{solar}^2 \sim 7 \times 10^{-5} \text{ eV}^2$ with large mixing angle $\tan^2 \theta_{solar} \sim 0.5$ [2], and the atmospheric ($\nu_\mu - \nu_\tau$) oscillations with $\Delta m_{atm}^2 \sim 2.5 \times 10^{-3} \text{ eV}^2$ and $\sin^2(2\theta) \simeq 1$ [3]. Within the SM all three Sakharov's conditions cannot be realized at the same time: baryon number violation, C and CP violation, and strong out of equilibrium condition [4]. Especially the last one is difficult to achieve with the Higgs mass constraint $\geq 114 \text{ GeV}$ from the LEP experiment [5]. There is a small range of parameter space left within minimal supersymmetric Standard Model (MSSM) where the electroweak baryogenesis can still work with the lightest stop mass lighter than the top quark mass [6]. There are currently other popular schemes of baryogenesis such as MSSM flat direction induced baryogenesis (for a review see [7]). An interesting point of Affleck Dine baryogenesis is that it generates baryon-isocurvature fluctuations [8], which can be constrained from the cosmic microwave background (CMB) data.

On the other hand the observed light neutrino masses can be obtained naturally if the Majorana nature of light neutrinos is confirmed along with a see saw scale [9]. An advantage of this is leptogenesis via L or $B - L$ violation and its subsequent conversion to baryon asymmetry through active SM sphalerons: $10^{12} \text{ GeV} \geq T \geq 100 \text{ GeV}$ [10].

It is almost impossible to test leptogenesis in a model independent way, because of the uncertainties in the scale of leptogenesis and the appearance of CP phase participating in leptogenesis. Especially this phase need not be the same as that of the low energy CP phase in the left handed neutrino sector. In a 3×3 scheme, where there are 3 heavy right handed and 3 light neutrinos, there are 18 real parameters and 3 CP violating phases. It has been proven extremely hard to make any prediction [11]. Some progress were made with 2 heavy and 3 light neutrino species where there are now 8 real parameters and 3 CP violating phases [12].

In this paper we will not follow the conventional argu-

ment of testing leptogenesis via measuring CP phases, but we will look forward to cosmology and particularly the physics of the cosmic microwave background radiation. In this regard we need two minimal assumptions: embedding leptogenesis in a supersymmetric set up and assume inflation. Both supersymmetry and inflation are necessary in their own rights. The advantage of supersymmetry is that it naturally provides a scalar component of the right handed neutrino field, sneutrino. On the other hand inflation creates a condensate for the sneutrino field with a non-vanishing vacuum expectation value (vev). If the lightest sneutrino mass is smaller than the Hubble expansion, then during inflation the quantum perturbations are stretched outside the horizon. The fermions also fluctuates during inflation, however they cannot be treated as a condensate due to lack of large occupation number. For earlier discussions on sneutrino induced leptogenesis, see [13–15]. Our discussion differs from that of Ref. [13,15] in some respects. We will always assume that the sneutrino condensate is not responsible for inflation, e.g. the inflaton energy density dominates over the sneutrino energy density, we will explain why we require so.

II. THERMAL VS NON-THERMAL LEPTOGENESIS

Leptogenesis can be thermal or non-thermal. In a thermal case the asymmetry is given by, for a review see [16],

$$\frac{n_B}{s} \approx \frac{8}{15} \frac{\epsilon_1}{g_*} \times \kappa, \quad (1)$$

where s is the entropy, the numerical factor accounts for the lepton baryon asymmetry in MSSM with two Higgs doublet, ϵ_1 is the CP asymmetry of the lightest right handed neutrino, and $g_* \sim \mathcal{O}(100)$ is the relativistic degrees of freedom. $\kappa \sim 10^{-1} - 10^{-2}$ is a measure of dilution estimated numerically by solving the Boltzmann equation for $\Delta L = 1$, $\Delta L = 2$ washout processes [17]. Therefore yielding $n_B/s \sim (10^{-3} - 10^{-4})\epsilon_1$.

On the other hand in the non-thermal leptogenesis the net asymmetry usually depends on the temperature of the decaying particles. If the lepton asymmetry is created before or during the inflaton decay, such that $\Gamma_N \geq \Gamma_X$, then the net baryon asymmetry is given by

$$\frac{n_B}{s} \approx \epsilon_1 \frac{T_{rh}}{m_\phi}, \quad (2)$$

where T_{rh} is the reheat temperature of the Universe and m_ϕ is the inflaton mass, the ratio of two arises due to entropy generation from the inflaton decay. For an example, for $T_{rh} \sim 10^9$ GeV, in order not to over produce thermal [18] and non-thermal gravitinos [19], and $m_\phi \sim 10^{13}$ GeV, in the case of chaotic inflation, then the net baryon asymmetry can be given by $n_B/s \sim 10^{-4}\epsilon_1$. Note that comparatively small $\epsilon_1 \leq 10^{-6}$ is required to yield a net baryon asymmetry.

However if the baryon asymmetry is solely created from the decay of the right handed neutrinos such that $\Gamma_N \leq \Gamma_X$, then the baryon asymmetry is given by

$$\frac{n_B}{s} \approx \epsilon_1 \gamma \frac{T_N}{m_N}, \quad (3)$$

where m_N is the right handed neutrino mass and γ accounts for the possible dilution of the asymmetry due to the entropy generation during reheating.

The advantages of thermal leptogenesis is that it requires minimal parameters, just CP asymmetry, e.g. ϵ_1 , while non-thermal leptogenesis undergoes the uncertainties of thermalization. Nevertheless non-thermal leptogenesis is inevitable in a supersymmetric context, as we argued earlier. The off-shoot of inflation is the formation of the lightest sneutrino condensate, and if this condensate survives, e.g. thermal scattering and evaporation, then the sneutrino induced leptogenesis is a rather natural phenomenon.

The greatest advantage of non-thermal leptogenesis is that it is testable from CMB, because the fluctuations in the sneutrino condensate can be transferred into the fluctuations in the baryon asymmetry, which gives rise to the baryon-isocurvature fluctuations. It is easy to see, where the reheat temperature obtains spatial fluctuations, the baryon asymmetry $\eta = (n_B/s)$ also obtains the large scale fluctuations,

$$\frac{\delta\eta}{\eta} \sim \mathcal{O}(1) \frac{\delta T_{rh}}{T_{rh}}. \quad (4)$$

On contrary thermal leptogenesis can never be tested in this way. Some critics may ponder on the feasibility of testing non-thermal leptogenesis via isocurvature fluctuations, because there could be many sources generating isocurvature fluctuations in the early Universe. The most popular paradigm could be the cold dark matter (CDM) isocurvature fluctuations, nevertheless within SUSY, thermal generation of CDM is likely to happen. Here we will rather take an opportunistic view point with the possibility of constraining right handed neutrino mass scale from CMB, which is an interesting topic in its own right.

III. TWO MODELS FOR NEUTRINO MASSES

The neutrinos obtain masses via Yukawa couplings from the Higgs vev. The Higgs field couples to the inflaton via a superpotential term, see for example [20],

$$W = \lambda X \Phi \bar{\Phi} + g \frac{\Phi \bar{\Phi}}{M_*} \mathbf{N} \mathbf{N} + h \mathbf{N} \mathbf{H}_u \mathbf{L}, \quad (5)$$

where X is a gauge singlet inflaton, Φ is the Higgs superfield, g, h are the Yukawas, \mathbf{N} is the right handed neutrino superfield, and M_* is the fundamental cut off of the theory, which could be either the string or the Planck scale $M_p = 2.4 \times 10^{18}$ GeV. The right handed neutrinos obtain mass from the vev v_ϕ after the end of inflation, which is given by (we will call this kind of models as type I model),

$$M_N \propto g \frac{v_\phi^2}{M_p}, \quad (6)$$

where g is a 3×3 matrix, here we have ignored the texture of the right handed neutrino mass matrix and we always assume a diagonal basis for the right handed neutrino mass matrix. The light neutrinos obtain masses via see-saw mechanism $m_\nu \sim m_D^2/M_N$, where m_D is the Dirac mass. For $v_\phi \sim 10^{15}$ GeV, the scale for the right handed neutrino masses comes out to be around 10^{12} GeV. Note that with the above superpotential term the right handed neutrino masses are identically zero during inflation, because X being an inflaton is only rolling down the potential while the Higgs is settled in its minimum, e.g. $\Phi, \bar{\Phi} = 0$. Inflation is supported by the Higgs vev, e.g. $V_{inf} \sim \lambda^2 v_\phi^4$.

There is also a non-renormalizable superpotential for the right handed neutrino field, which is valid below the $U(1)_{B-L}$ breaking scale or the $SO(10)$ breaking scale, v_ϕ ,

$$W = \lambda_1 \frac{N^n}{n M_p^{n-3}}. \quad (7)$$

If R -parity is conserved then the right handed neutrino direction is lifted by $n = 4$ operator, and $|\lambda_1| \leq \mathcal{O}(1)$ is treated as a free parameter. Besides this correction there are various soft SUSY breaking terms, such as $(m_{3/2}^2 + C_N H^2) \tilde{N}^2$, where $m_{3/2}$ is the gravitino mass. The A -terms, $(a_1 m_{3/2} \tilde{N} \tilde{h}_u \tilde{l} + a_2 H \tilde{N} \tilde{h}_u \tilde{l})$, and the non-renormalizable potential for the sneutrino, $(a_3 H/M_p^{n-3} \tilde{N}^n + a_4 m_{3/2}/M_p^{n-3} \tilde{N}^n + \text{h.c.})$, where $|a_i| \sim \mathcal{O}(1)$ are complex numbers, and tilde denotes sparticle. We always consider $n = 4$ in our example. The Hubble induced A -terms and the Hubble induced soft SUSY breaking mass terms are possible if the inflaton potential, v_ϕ , arises from the F -sector. However in the D -term inflation case the Hubble induced mass and the Hubble induced A -term correction does not arise.

Note that if we embed the right handed neutrino sector into a gauge group, e.g. $SO(10)$, then we would also

expect the D-term contributions for \tilde{N} . However if the vev of \tilde{N} is less than the $SO(10)$ breaking scale then the D-term contribution decouples from the rest of the potential [13].

For the sake of illustration let us consider a D-term inflation. The scalar potential during inflation is then given by

$$V \approx \lambda^2 v_\phi^4 + \frac{\lambda^4 v_\phi^4}{16\pi^2} \ln\left(\frac{X}{M_p}\right) + \lambda_1^2 \frac{\tilde{N}^6}{M_p^2}. \quad (8)$$

The second term in the above potential is the one-loop Coleman-Weinberg contribution due to SUSY breaking. Note that we have neglected the soft SUSY breaking contributions here. At sufficiently large scales it is the non-renormalizable term dominates the potential. For simplicity and for the purpose of illustration we assume that the lightest right handed electron sneutrino forms a condensate.

In this paper we would like to advocate that the inflaton sector always dominate the sneutrino sector. There have been some suggestions regarding sneutrino dominated inflation, but there are problems associated with that, e.g. setting up a gauged sector for the sneutrino will be impossible, especially when the sneutrino vev is larger than M_p . In this case the inflaton vev, X , can be related to the number of e-foldings of inflation, \mathcal{N}_e , e.g.

$$X \approx \lambda \frac{M_p}{2\pi} \sqrt{\mathcal{N}_e}. \quad (9)$$

The initial vev of \tilde{N} has to be smaller than the critical value, in order not to ruin the inflation, which is given by

$$\tilde{N}_c = \sqrt{2} \left(\frac{\lambda}{\sqrt{2}\lambda_1} v_\phi^2 M_p \right)^{1/3}. \quad (10)$$

We note that in this class of models the slow roll conditions are governed by the inflaton, $\epsilon_X \ll |\eta_X| \approx M_p^2 V(X)''/V_0 \approx 1/(2\mathcal{N}_e) \ll 1$, where $V_0 = \lambda^2 v_\phi^4$. The spectral index is given by

$$n-1 \approx -3M_p^2 \frac{V'(X)^2}{V_0^2} + 2M_p^2 \frac{V''(X)}{V_0} - M_p^2 \frac{V'(\tilde{N})^2}{V_0^2} 2\eta_X \sim 1/\mathcal{N}_e. \quad (11)$$

The right handed neutrinos may also obtain masses which need not have any connection to the inflaton sector. If this be the case then the superpotential for the right handed neutrino sector can be written as [21,22]

$$W = \frac{1}{2} g \mathbf{X} \mathbf{N} \mathbf{N} + h \mathbf{N} \mathbf{H}_u \mathbf{L} + \frac{1}{2} M_N \mathbf{N} \mathbf{N}, \quad (12)$$

where g , h are the Yukawas, and M_N is the right handed neutrino mass term, which breaks the lepton number. We work on a diagonal basis for the right handed neutrinos and we assume that the texture is such that the lightest

right handed neutrino mass is larger than the reheat temperature. We call this type of model as type *II*, because the masses of the right handed neutrinos are completely independent of the inflaton sector.

In the above superpotential note that we have an explicit coupling between \mathbf{X} and \mathbf{N} . This coupling is not absolutely necessary, but X being a SM gauge singlet can couple to the right handed neutrino sector. The above superpotential has an advantage that the inflaton can decay via the right handed neutrino sector (off-shell or on-shell) to the Higgs and the lepton doublet. Therefore reheating the Universe with the SM degrees of freedom or more precisely twice the SM relativistic degrees of freedom. Reheating naturally provides the way out of equilibrium condition.

IV. DENSITY PERTURBATIONS FROM SNEUTRINO

In fact one can also imagine that the Yukawa coupling, g , in Eq. (12) has a non-renormalizable contribution of the form [22]

$$g = g_0 \left(1 + \frac{\mathbf{N}}{M_p} + \dots \right). \quad (13)$$

Such a coupling can be easily accommodated at the level of superpotential.

If the sneutrino field is light enough compared to the Hubble expansion during inflation, e.g. the lightest of the sneutrino field, then the perturbations generated in the sneutrino field can seed perturbations in the inflaton sector. The sneutrino fluctuations are isocurvature in nature, which are converted into the adiabatic fluctuations at the time of reheating.

Note that in this case the inflaton coupling to the matter field, g , is fluctuating. The reheat temperature, which is given by the inflaton coupling to the right handed (s)neutrino field, $T_{rh} \sim g \sqrt{m_X M_p}$, also fluctuates. Since the energy density stored in the relativistic species is $\rho_r \propto T_{rh}^4$, therefore during inflaton dominated oscillations the ratio of energy densities at two different times is given by

$$\frac{\rho_2}{\rho_1} = \left(\frac{g_1}{g_2} \right)^{4/3}. \quad (14)$$

The factor 4/3 appears due to red-shift of the scale factor, which is during inflaton oscillations following $a(t) \propto t^{2/3}$ [23–25]. This gives rise to the fluctuations in the energy density which is finally imprinted upon CMB. The fluctuation in the energy density of the relativistic species is given by [23–25]

$$\frac{\delta\rho}{\rho} = -\frac{4}{3} \frac{\delta g}{g} = -\frac{4}{3} \frac{\delta N}{N}. \quad (15)$$

Yet another useful way of imagining the coupling term g in Eq. (13) as a fluctuating mass term for the inflaton X .

A. Multi-field perturbations

The perturbations are defined on a finite energy density hypersurface foliated in a coordinate system such that the metric perturbation is ζ , and the metric (for a detailed discussion on cosmological density perturbations, see [26]) is given by

$$ds^2 = a^2(t) (1 + 2\zeta) \delta_{ij} dx^i dx^j, \quad (16)$$

where a is the scale factor. The time evolution of the curvature perturbation, ζ , on scales larger than the size of the horizon is given by [27–29]

$$\dot{\zeta} = -\frac{H}{\rho + P} \delta P_{\text{nad}}, \quad (17)$$

where $P_{\text{nad}} \equiv \delta P - c_s^2 \delta \rho$ is the non-adiabatic pressure perturbation. The adiabatic sound speed is $c_s^2 = \dot{P}/\dot{\rho}$, where P and ρ are the total pressure and the energy density. For a single field $\delta P_{\text{nad}} = 0$, therefore on large scales the curvature perturbation is pure adiabatic in nature with $\zeta = \text{constant}$. This is not true in presence of many fields, because the relative pressure perturbations between fields can give non-zero contribution to δP_{nad} .

The total curvature perturbation Eq. (17), for many fields, can be written in terms of various components [28,29]

$$\zeta = \sum_{\alpha} \frac{\dot{\rho}_{\alpha}}{\dot{\rho}} \zeta_{\alpha}. \quad (18)$$

Isocurvature or entropy perturbations describe the difference between the curvature perturbations [28]

$$S_{\alpha\beta} = 3(\zeta_{\alpha} - \zeta_{\beta}) = -3H \left(\frac{\delta \rho_{\alpha}}{\dot{\rho}_{\alpha}} - \frac{\delta \rho_{\beta}}{\dot{\rho}_{\beta}} \right). \quad (19)$$

With the help of Eqs. (18,19), we obtain a useful relationship connecting the curvature and the entropy perturbations

$$\zeta_{\alpha} = \zeta + \frac{1}{3} \sum_{\beta} \frac{\dot{\rho}_{\beta}}{\dot{\rho}} S_{\alpha\beta}. \quad (20)$$

An important quantity is the gauge invariant comoving curvature perturbations which is defined as

$$\mathcal{R} = H \sum_{\alpha} \left(\frac{\dot{\phi}_{\alpha}}{\sum_{\beta} \dot{\phi}_{\beta}^2} \right) Q_{\alpha}, \quad (21)$$

where Q_{α} is the Sasaki-Mukhanov variable defined in terms of the gauge invariant quantities

$$Q_{\alpha} \equiv \delta \phi_i + \frac{\dot{\phi}_i}{H} \psi. \quad (22)$$

where ψ is related to the curvature perturbations by

$$\psi = -\zeta + H \frac{\delta \rho}{\dot{\rho}}. \quad (23)$$

The comoving curvature perturbation is defined in terms of the Sasaki-Mukhanov variable [29]

$$\mathcal{R}_{\alpha} \equiv \psi + \frac{H}{\dot{\phi}_{\alpha}} Q_{\alpha}. \quad (24)$$

The comoving curvature perturbation is dominated by the field with a dominating kinetic term.

There is another useful gauge invariant combination [28]

$$\delta_{\alpha\beta} \equiv \left(\frac{\delta \phi_{\alpha}}{\dot{\phi}_{\alpha}} - \frac{\delta \phi_{\beta}}{\dot{\phi}_{\beta}} \right), \quad (25)$$

and the isocurvature perturbations can be defined as [30]

$$S_{\alpha\beta} = a^3 \frac{d}{dt} \left(\frac{\delta_{\alpha\beta}}{a^3} \right). \quad (26)$$

We will use the above results in the coming sections.

Especially for two fields case it is fairly easy to investigate the adiabatic and the isocurvature fluctuations [31,29]. Our two fields are the inflaton and the sneutrino field. For the purpose of illustration, we assume there is a single sneutrino component responsible for the fluctuations, though in principle all three sneutrino components could feel the fluctuations. However the lightest among all will have a greater impact, or one can also assume that all three neutrinos are nearly degenerate. In which case a linear combination of all the species provide the entire perturbations.

We define the adiabatic component as σ and the entropic component by s , such that,

$$\delta \sigma = (\cos \theta) \delta X + (\sin \theta) \delta \tilde{N}, \quad (27)$$

$$\delta s = (\cos \theta) \delta \tilde{N} - (\sin \theta) \delta X, \quad (28)$$

where

$$\cos \theta = \dot{X} / \sqrt{\dot{X}^2 + \dot{\tilde{N}}^2}, \quad \sin \theta = \dot{\tilde{N}} / \sqrt{\dot{X}^2 + \dot{\tilde{N}}^2}. \quad (29)$$

Therefore

$$\delta s = \frac{\dot{X} \dot{\tilde{N}}}{\sqrt{\dot{X}^2 + \dot{\tilde{N}}^2}} \left(\frac{\delta \tilde{N}}{\dot{\tilde{N}}} - \frac{\delta X}{\dot{X}} \right) = \frac{\dot{X} \dot{\tilde{N}}}{\sqrt{\dot{X}^2 + \dot{\tilde{N}}^2}} \delta_{\tilde{N}X}. \quad (30)$$

The comoving perturbations, \mathcal{R} , can be calculated from Eq. (27) in a spatially flat gauge where $\psi = 0$,

$$\mathcal{R} \approx H \left(\frac{\dot{X} \delta X + \dot{\tilde{N}} \delta \tilde{N}}{\dot{X}^2 + \dot{\tilde{N}}^2} \right) = H \frac{\delta \sigma}{\dot{\sigma}}. \quad (31)$$

Here we assumed slow roll conditions. The entropy perturbations can be calculated by combining Eqs. (26,30).

$$S = -H \left(\frac{\delta \tilde{N}}{\tilde{N}} - \frac{\delta X}{\dot{X}} \right) = -H \frac{\sqrt{\dot{X}^2 + \dot{\tilde{N}}^2}}{\dot{X} \tilde{N}} \delta s. \quad (32)$$

For $\dot{X} \gg \dot{\tilde{N}}$, the above expression reduces to $S \approx H(\delta S/\tilde{N})$, which has some significance when dealing with the isocurvature fluctuations. Also note that when the perturbations in the inflaton is assumed to be small, such that $\delta X \ll \delta \tilde{N}$, or $\dot{X} \gg \delta X$, then the entire perturbations come from the sneutrino field, e.g. $S \approx -H(\delta \tilde{N}/\tilde{N})$. Note that when the perturbations in the inflaton X is neglected then the entropy perturbation arises from the sneutrino sector which is solely responsible for feeding the adiabatic mode. This is a special case which we will discuss later on.

Due to the random Gaussian vacuum fluctuations of X , \tilde{N} , the fields acquire a spectrum at the time of horizon crossing

$$\mathcal{P}_{\delta X}|_* \approx \mathcal{P}_{\delta \tilde{N}}|_* \approx \left(\frac{H_*}{2\pi} \right)^2, \quad (33)$$

with zero cross correlation, e.g. $\mathcal{C}_{\delta X, \delta \tilde{N}} = 0$, which we assume for simplicity. In terms of local rotations, δs , $\delta \sigma$, the spectrum is also proportional to $(H_*/2\pi)^2$. Therefore

$$\mathcal{P}_R|_* \approx \left(\frac{H^2}{2\pi \dot{\sigma}} \right)^2 \Big|_* \sim \frac{8}{3\epsilon} \frac{V_*}{M_p^4}. \quad (34)$$

where $\epsilon = \epsilon_X + \epsilon_{\tilde{N}}$. Similarly for the isocurvature fluctuations the spectrum is given by

$$\mathcal{P}_S|_* \approx \left(\frac{\dot{\sigma} H^2}{2\pi \dot{X} \dot{\tilde{N}}} \right)^2 \Big|_*. \quad (35)$$

This expression reduces to a simple form if $\dot{X} \gg \dot{\tilde{N}}$, then $\mathcal{P}_S|_* \approx \left(H^2/2\pi \dot{\tilde{N}} \right)^2 \Big|_*$.

V. SOME EXAMPLES

A. Isocurvature fluctuations in type I scenarios

During inflation the sneutrino condensate evolves in the non-renormalizable potential given by Eq. (8). In addition, we assume that there is no Hubble-induced mass term for the sneutrino even after inflation, so that the field will continue slow-rolling in the non-renormalizable potential down to the amplitude \tilde{N}_{osc} , when its energy density is determined by $V(\tilde{N}_{osc}) \sim M_N^2 \tilde{N}_{osc}^2$. In general, the equations of motion for the homogeneous and the fluctuation parts are written by

$$\ddot{\tilde{N}} + 3H\dot{\tilde{N}} + V'(\tilde{N}) = 0, \quad (36)$$

$$\delta \ddot{\tilde{N}}_k + 3H\delta \dot{\tilde{N}}_k + \frac{k^2}{a^2} \delta \tilde{N}_k + V''(\tilde{N}) \delta \tilde{N}_k = 0. \quad (37)$$

Since we are interested only in the super horizon mode ($k \rightarrow 0$), using the slow roll approximations we have

$$3H\dot{\tilde{N}} + V'(\tilde{N}) = 0, \quad (38)$$

$$3H\delta \dot{\tilde{N}} + V''(\tilde{N}) \delta \tilde{N} = 0. \quad (39)$$

Hereafter we omit the subscript k , understanding that $\delta \tilde{N}$ is for the super horizon mode. Then it is easy to obtain the evolution of the ratio of the fluctuation and the homogeneous mode in a $V_{NR} \propto \tilde{N}^6$ potential. The result is [24]

$$\frac{\delta \tilde{N}}{\tilde{N}} \sim \left(\frac{\delta \tilde{N}}{\tilde{N}} \right)_i \left(\frac{\tilde{N}}{\tilde{N}_i} \right)^4, \quad (40)$$

where i denotes the initial values.

During inflation the homogeneous field obeys Eq. (38), which can be easily integrated to yield

$$\frac{\tilde{N}}{\tilde{N}_i} \simeq \left(1 + \frac{1}{15} \frac{V''(\tilde{N}_i)}{H^2} \mathcal{N}_e \right)^{-\frac{1}{4}}. \quad (41)$$

Since we are concerned within a slow-roll regime, it is reasonable to require $V''(\tilde{N}_i)/H^2 \lesssim 1$. Hence we have, $\tilde{N}/\tilde{N}_i \approx 0.67$, for the last 60 e-folds in this case. This implies that the amplitude of the fluctuation relative to its homogeneous part decreases only by a factor $\simeq 0.2$. Hence during this stage there is less damping. Notice that slower the condensate field rolls during the last 60 e-folds, the less damping there is.

After inflation the sneutrino condensate slow-rolls (albeit marginally), i.e., $V''(\tilde{N}) \sim H^2$, and we can still use the slow-roll approximations, e.g. Eqs. (38,39). This will give the largest estimate on the dilution of the amplitude. During this stage the field amplitude is given by $\tilde{N} \sim (HM_p/\lambda)^{1/2}$, while the Hubble parameter changes from H_* to $M_{\tilde{N}}$. As a consequence, there is a damping given by

$$\frac{\left(\frac{\delta \tilde{N}}{\tilde{N}} \right)_{osc}}{\left(\frac{\delta \tilde{N}}{\tilde{N}} \right)_*} \sim \left(\frac{M_{\tilde{N}}}{H_*} \right)^2 \sim 3 \times g^2. \quad (42)$$

The last equality follows from Eq. (6).

At the time when the sneutrino decays, e.g. $H \equiv \Gamma_N \sim h^2 M_N/4\pi$, where the largest Yukawa coupling is that of the tau doublet, of the order of $h \sim \mathcal{O}(10^{-4})$, which takes place when the net lepton number is given by $n_L = \rho_{\tilde{N}}/M_{\tilde{N}} \sim M_{\tilde{N}} \tilde{N}^2$. Therefore the isocurvature perturbations are given by

$$S = \frac{\delta n_L}{n_L} \frac{\Omega_B}{\Omega_m} = \frac{\delta n_B}{n_B} \frac{\Omega_B}{\Omega_m} \approx 6g^2 \left(\frac{\delta \tilde{N}}{\tilde{N}} \right)_* \frac{\Omega_B}{\Omega_m} \approx 6g^2 \frac{H_*}{2\pi \tilde{N}_*} \frac{\Omega_B}{\Omega_m}. \quad (43)$$

The subscript m denotes total matter density. To compare the two types of fluctuations, it may be useful to consider the ratio between the adiabatic and the isocurvature

$$\alpha \sim \frac{\mathcal{P}_S^{1/2}}{\mathcal{P}_R^{1/2}} \Big|_* \approx \frac{3g^2\lambda^2}{32\sqrt{2}\pi^3} \frac{M_p^2}{X_*\tilde{N}_*} \left(\frac{\Omega_B}{\Omega_m} \right), \approx \frac{3g^2\lambda}{16\sqrt{2}\pi^2\sqrt{\mathcal{N}_e}} \frac{M_p}{\tilde{N}_*} \left(\frac{\Omega_B}{\Omega_m} \right). \quad (44)$$

The last equality comes due to Eq. (9). There are couple of points to be mentioned. First of all in type I model, we assumed that the major adiabatic fluctuations arose from the inflaton sector. The total power spectrum is given by $\mathcal{P} = \mathcal{P}_R + \mathcal{P}_S$. We also assumed that there is no correlation. Recent observations from WMAP data constraints this ratio [32]. The uncorrelated isocurvature fluctuations in the CDM has been presented in a recent analysis, which suggests the ratio $\alpha < 0.31$ at 95% c.l. If we take $\mathcal{N}_e = 50$, $\Omega_B h^2 = 0.023$, $\Omega_m h^2 = 0.133$, we find

$$\tilde{N}_* > 10^{-3} g^2 \lambda M_p. \quad (45)$$

On the other hand, in order to have inflation and the spectral index governed solely by the inflaton field, $\tilde{N} < \tilde{N}_c$, from Eq. (10), which puts additional constraints on model parameters such as g , λ , λ_1 , v_ϕ , or in terms of the right handed neutrino mass scale M_N , the constraints translate to

$$M_N \geq 5 \times 10^{-10} g^7 \lambda^2 \lambda_1 M_p. \quad (46)$$

Note that the scale of the right handed neutrino is quite sensitive to the Yukawa coupling g . For the couplings order one, we obtain a reasonable bound $M_N \geq 5 \times 10^8$ GeV. Also note that if we assume that the right handed neutrino sector is embedded in a gauge sector such as GUT, the $\tilde{N}_* \leq 10^{16.5}$ GeV, assuming that the GUT scale is at $10^{16.5}$ GeV, suggesting that the Yukawas, $g^2 \lambda \ll \mathcal{O}(1)$. Additional constraints will resurface based on thermal history of the Universe during inflaton decay, e.g. sneutrino interacting in a finite temperature thermal bath. Some of these issues are model dependent, such as how inflaton is reheating the Universe, whether the inflaton is decaying predominantly into the MSSM degrees of freedom or some other, etc. Hopefully we will address them in a future publication. Strictly speaking the bounds Eqs. (45,46) hold true only if the finite temperature effects are negligible.

B. Isocurvature fluctuations in type II scenarios

In type II scenario there is no need for the explicit inflaton coupling to the right handed neutrino sector. In fact if $g = 0$ in Eq. (12), then the sneutrino evolution will be similar to what we already discussed in our previous scenario. The fluctuations in the inflaton and the sneutrino sector could be treated separately on scales larger than the size of the horizon. Even in this case one can expect non-renormalizable superpotential contribution to the sneutrino. Nevertheless if the initial amplitude for the sneutrino, e.g. $\tilde{N}_i \leq (M_N M_p / 2\lambda_1)^{1/2}$, then the non-renormalizable contribution will not play any significant role, and therefore there will be no damping in the amplitude of the fluctuations in the sneutrino sector after the end of inflation. The above estimation for the baryon isocurvature fluctuations in Eq. (43) holds true without the damping factor $6g^2$. As a result the sneutrino vev must have

$$\tilde{N}_* > 10^{-4} \lambda M_p, \quad (47)$$

assuming that the inflaton sector is given by $V(X)$, see Eq. (8). The bound on the neutrino mass scale arises from the dominance of the inflaton energy density,

$$M_N < 10^4 \frac{v_\phi^2}{M_p}, \quad (48)$$

over the sneutrino condensate.

The non vanishing inflaton right handed neutrino coupling gives rise to a completely new feature. The isocurvature fluctuations in this case will be certainly correlated, and there is a new possibility that the sneutrino induced isocurvature fluctuations get converted into the adiabatic fluctuations. We will discuss this issue later on in a separate subsection.

C. What if $M_N \geq H_{inf}$?

It is quite possible that the right handed neutrino mass scale is greater than or equal to the Hubble expansion during inflation. In either case the sneutrino will roll down to its minimum from its initial vev in less than one Hubble time and starts oscillating before settling down with a vanishing kinetic term. If the Yukawa coupling h is sufficiently large, $\Gamma_N \gg H_{inf} \gg \Gamma_X$, then the sneutrino might even decay during inflation. In an opposite limit $\Gamma_N \leq H_{inf}$, the sneutrino survives inflation, and feels the quantum fluctuations.

The solutions for Eq. (37) with $V(\tilde{N}) = \alpha H_{inf}^2 \tilde{N}^2$ is well known [33] (from here onwards we drop the subscript *inf*)

$$\delta \tilde{N}_k \approx H (H a)^{-3/2} \left(\frac{k}{aH} \right)^{-\sqrt{9/4-\alpha}}, \quad (49)$$

and the power spectrum follows

$$\mathcal{P}_{\delta\tilde{N}} \propto H^{3/2} \left(\frac{k}{aH} \right)^{3/2 - \text{Re}(\sqrt{9/4 - \alpha})}. \quad (50)$$

For $\alpha \gg 9/4$ the real part of the exponent vanishes leaving a very steep spectrum for the isocurvature perturbations, e.g. $\mathcal{P}_S \propto k^3$, which is exponentially suppressed by the end of inflation. This regime certainly has not much of interest. However if $\alpha \leq 9/4$ then $\mathcal{P}_S \propto k^{2\alpha/3}$ [33]. Though in this case the tilt in the isocurvature power spectrum plays an important role in constraining

$$n_{iso} = \frac{4\alpha}{3}. \quad (51)$$

For the uncorrelated isocurvature fluctuations, the constraint on the isocurvature spectral index is $n_{iso} = 1.02$ at 95% c.l. [34]. This gives a limit on the right handed neutrino mass scale, $M_N \sim 0.7H$.

D. Correlated baryon-Isocurvature fluctuations

Most of the examples belong to the category where the isocurvature and the adiabatic fluctuations are uncorrelated, because the two fields had fluctuations independent of each other $\langle R_* S_* \rangle = 0$. Here we consider a simple example $\langle R_* S_* \rangle \neq 0$. We already set up a superpotential term Eq. (12) with a coupling Eq. (13).

Following Eqs. (14, 15), we notice that the fluctuation in the reheat temperature is given by

$$\frac{\delta T_{rh}}{T_{rh}} = -\frac{1}{3} \frac{\delta g}{g} \sim -\frac{\delta \tilde{N}_*}{3M_p} \sim -\frac{H_*}{6\pi M_p}. \quad (52)$$

An important point to note is that the baryon asymmetry is also proportional to g , see Ref. [22]. Therefore baryons also feel the spatial fluctuations.

$$\frac{\delta \eta_B}{\eta_B} \sim -\frac{1}{3} \frac{\delta g}{g} \sim -\frac{\delta \tilde{N}}{3M_p} \sim \frac{\delta T_R}{T_R} \neq 0. \quad (53)$$

The origin of $-1/3$ factor has a similar origin as Eq. (52). Note that the fluctuations in the baryon asymmetry is proportional to the fluctuations in the inflaton coupling, and therefore fluctuations in the reheat temperature. This shows that the baryonic asymmetry does not follow the adiabatic density perturbations, instead the perturbation in the baryons is correlated baryon-isocurvature in nature. The two fluctuations; isocurvature and adiabatic perturbations are not independent of each other. Rather the former feeds the latter ones.

The baryon-isocurvature fluctuations leaves its imprint upon the cosmic micro wave background radiation.

$$S_B = \frac{\delta \eta_B}{\eta_B} = \frac{\delta T_{rh}}{T_{rh}}, \quad (54)$$

$$\zeta = -H \frac{\delta \rho_\gamma}{\dot{\rho}_\gamma} = \frac{1}{4} \frac{\delta \rho_\gamma}{\rho_\gamma} = \frac{\delta T_{rh}}{T_{rh}}, \quad (55)$$

where the subscript γ denotes MS(SM) radiation. Therefore we find $|S_B/\zeta| = 1$. This toy model has a unique prediction which can be ruled out easily from the future cosmic microwave background experiments.

VI. CONCLUSION

We argued that the non-thermal leptogenesis is potentially testable from its contribution to the baryon isocurvature fluctuations. There could be many other sources for the isocurvature perturbations during inflation including the most competitive candidate “cold dark matter”. However within SUSY excellent conditions arise naturally for a thermal production of the CDM. We also note that the thermal leptogenesis is a viable scheme, nevertheless, it is quite natural that during inflation the sneutrino condensate can be created. If the condensate survives inflation and the thermal bath created by the inflaton decay products, then the sneutrino decay can generate the lepton asymmetry. The asymmetry created in the lepton sector inherits the spatial fluctuations from the sneutrinos during inflation, which are isocurvature in nature. These isocurvature fluctuations could be uncorrelated and/or correlated in nature. In this paper we have provided examples which are tied up with the inflation sector, and we have also given examples of correlated and uncorrelated isocurvature fluctuations. We estimated the ratio S/R , which constrains various model parameters and also the mass scale of the right handed neutrinos. In the simplest realization of type *I* leptogenesis, we found the lightest sneutrino vev to be $\tilde{N} > 10^{-3} g^2 \lambda M_p$, and $M_N \geq 5 \times 10^{-10} g^7 \lambda^2 \lambda_1 M_p$. In type *II* case, $\tilde{N}_* > 10^{-4} \lambda M_p$, and $M_N < 10^4 v_\phi^2 / M_p$. We also noticed that when the sneutrino mass is heavier than the Hubble expansion during inflation, then the isocurvature perturbations die away. However if their mass ranges are close to the Hubble expansion during inflation, then the important constraint arises from the spectral tilt $n_{iso} = (4M_N^2/3H_*^2)$. We also gave an example of a toy model where the perturbations are correlated and there is a unique prediction for $S/R = 1$.

Though, for simplicity we restricted our perturbation analysis to the two fields, in reality an elaborate treatment of the perturbations of all three generations of the sneutrinos is necessary, nevertheless, we catch an interesting glimpse of the problem. In principle the formalism developed here can be carried on to incorporating all three generations, which we leave for future investigation.

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